

pst-vehicle v 1.2

A PSTricks package for slipping/rolling vehicles on curves of any kind of mathematical functions

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This package was created to illustrate the notion of slope, the coefficient of the tangent line at a point of a curve. On the road, a rampant way or a dangerous descent due to their slope is indicated by a sign showing the percentage of the slope of this section of road, for example 10%. It was therefore quite obvious that the idea of representing a vehicle rolling without slipping on a curve came into our minds. Different types of vehicles are proposed, the shape of the curve is to be defined by its equation: y = f(x) in algebraic notation. The line connecting the two contact points from the front and the rear wheel with the curve and the sign of the slope can be easily displayed. It is also possible to represent, not a speed-o-meter of the vehicle, but a slope-o-meter was introduced as an indicator of the value of the slope of the straight line defined above.

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1 Theory—the mathematical background

Within the following first sections, we like to show the theory on how we programmed the package. Easy elementary vector geometry with some basic calculus aspects.

1.1 Wheels on a curve—equal radii



The boundary point of the back wheel (radius r from the back wheel axis) with the curve:

$$\vec{x}_0 = \begin{pmatrix} x_0 \\ f(x_0) \end{pmatrix}$$

The direction of the tangent line is:

$$\vec{t}_0 = \begin{pmatrix} 1\\ f'(x_0) \end{pmatrix}$$

The normed normal vector in x_0 is:

$$\vec{n}_{0x_0} = \frac{1}{\sqrt{1 + f'(x_0)^2}} \begin{pmatrix} -f'(x_0) \\ 1 \end{pmatrix}$$

The point H – coordinates of the back wheel axis is:

$$\overline{BW} = \vec{x}_0 + r \cdot \vec{n}_{0x_0}$$
$$= \begin{pmatrix} x_0 - r \frac{f'(x_0)}{\sqrt{1 + f'(x_0)^2}} \\ f(x_0) + r \frac{1}{\sqrt{1 + f'(x_0)^2}} \end{pmatrix}$$

x is the abscissa of the boundary point of the front wheel with the curve. The normed normal vector in x is:

$$\vec{n}_{0x} = \frac{1}{\sqrt{1 + f'(x)^2}} \begin{pmatrix} -f'(x) \\ 1 \end{pmatrix}$$

The point V – coordinates of the front wheel axis is:

$$\overline{FW} = \vec{x} + r \cdot \vec{n}_{0x} \\ = \begin{pmatrix} x - r \frac{f'(x)}{\sqrt{1 + f'(x)^2}} \\ f(x) + r \frac{1}{\sqrt{1 + f'(x)^2}} \end{pmatrix}$$

ς.

The distance between the axes is R – thus:

$$|\overrightarrow{FW} - \overrightarrow{BW}| = R$$

$$\left| \begin{pmatrix} x - r\frac{f'(x)}{\sqrt{1 + f'(x)^2}} - \left(x_0 - r\frac{f'(x_0)}{\sqrt{1 + f'(x_0)^2}}\right) \\ f(x) + r\frac{1}{\sqrt{1 + f'(x)^2}} - \left(f(x_0) + r\frac{1}{\sqrt{1 + f'(x_0)^2}}\right) \end{pmatrix} \right| = R$$

$$\left| \begin{pmatrix} x - x_0 + r\left(\frac{f'(x_0)}{\sqrt{1 + f'(x_0)^2}} - \frac{f'(x)}{\sqrt{1 + f'(x_0)^2}}\right) \\ f(x) - f(x_0) + r\left(\frac{1}{\sqrt{1 + f'(x)^2}} - \frac{1}{\sqrt{1 + f'(x_0)^2}}\right) \end{pmatrix} \right| = R$$

This leads to an equation of the variable x, where x is the abscissa of the boundary point of the front wheel with the curve.

$$\left(x - x_0 + r\left(\frac{f'(x_0)}{\sqrt{1 + f'(x_0)^2}} - \frac{f'(x)}{\sqrt{1 + f'(x)^2}}\right)\right)^2 + \left(f(x) - f(x_0) + r\left(\frac{1}{\sqrt{1 + f'(x)^2}} - \frac{1}{\sqrt{1 + f'(x_0)^2}}\right)\right)^2 = R^2$$

1.2 Wheels on a curve—different radii



The boundary point of the back wheel (radius r_1 from the back wheel axis) with the curve:

$$\vec{x}_0 = \begin{pmatrix} x_0 \\ f(x_0) \end{pmatrix}$$

The direction of the tangent line is:

$$\vec{t}_0 = \begin{pmatrix} 1\\ f'(x_0) \end{pmatrix}$$

The normed normal vector in x_0 is:

$$\vec{n}_{0x_0} = \frac{1}{\sqrt{1 + f'(x_0)^2}} \begin{pmatrix} -f'(x_0) \\ 1 \end{pmatrix}$$

The point H – coordinates of the back wheel axis is:

$$\overrightarrow{BW} = \vec{x}_0 + r_1 \cdot \vec{n}_{0x_0}$$
$$= \begin{pmatrix} x_0 - r_1 \frac{f'(x_0)}{\sqrt{1 + f'(x_0)^2}} \\ f(x_0) + r_1 \frac{1}{\sqrt{1 + f'(x_0)^2}} \end{pmatrix}$$

 \boldsymbol{x} is the abscissa of the boundary point of the front wheel with the curve. The normed normal vector in \boldsymbol{x} is:

$$\vec{n}_{0x} = \frac{1}{\sqrt{1 + f'(x)^2}} \begin{pmatrix} -f'(x) \\ 1 \end{pmatrix}$$

The point V – coordinates of the front wheel axis is:

$$\overline{FW} = \vec{x} + r_2 \cdot \vec{n}_{0x}$$
$$= \begin{pmatrix} x - r_2 \frac{f'(x)}{\sqrt{1 + f'(x)^2}} \\ f(x) + r_2 \frac{1}{\sqrt{1 + f'(x)^2}} \end{pmatrix}$$

The distance between the axes is ${\it R}$ – thus:

$$|\overrightarrow{FW} - \overrightarrow{BW}| = R$$

$$\left| \begin{pmatrix} x - r_2 \frac{f'(x)}{\sqrt{1 + f'(x)^2}} - \begin{pmatrix} x_0 - r_1 \frac{f'(x_0)}{\sqrt{1 + f'(x_0)^2}} \end{pmatrix} \\ f(x) + r_2 \frac{1}{\sqrt{1 + f'(x)^2}} - \begin{pmatrix} f(x_0) + r_2 \frac{1}{\sqrt{1 + f'(x_0)^2}} \end{pmatrix} \end{pmatrix} \right| = R$$

This leads to an equation of the variable x, where x is the abscissa of the boundary point of the front wheel with the curve.

$$\left(x - x_0 + r_1 \frac{f'(x_0)}{\sqrt{1 + f'(x_0)^2}} - r_2 \frac{f'(x)}{\sqrt{1 + f'(x)^2}}\right)^2 + \left(f(x) - f(x_0) + r_2 \frac{1}{\sqrt{1 + f'(x)^2}} - r_1 \frac{1}{\sqrt{1 + f'(x_0)^2}}\right)^2 = R^2$$

1.3 Inclination angle between back wheel and front wheel axes



The initial inclination angle α of the line between front wheel axis and back wheel axis on a horizon-

tal plane is :

$$\alpha = \arctan\left(\frac{r_1 - r_2}{\sqrt{R^2 - (r_1 - r_2)^2}}\right)$$

If the plane is not horizontal, there is an additional angle β given by the function f(x):



The coordinates of the back wheel axis are:

$$\overrightarrow{BW} = \vec{x}_0 + r_1 \cdot \vec{n}_{0x_0} = \begin{pmatrix} x_0 - r_1 \frac{f'(x_0)}{\sqrt{1 + f'(x_0)^2}} \\ f(x_0) + r_1 \frac{1}{\sqrt{1 + f'(x_0)^2}} \end{pmatrix} = \begin{pmatrix} x_H \\ y_H \end{pmatrix}$$

The angle β is:

$$\beta = \arctan\left(\frac{y_V - y_H}{x_V - x_H}\right)$$

The total angle γ is:

$$\gamma = -(\alpha + \beta)$$

1.4 Determination of the curvature radius

A curved curve can be imagined from many small circular arcs. The radius of the respective associated circles is referred to as the radius of curvature. The stronger the curvature of a curve changes, the smaller the intervals have to be chosen in order to be able to speak approximately of a circular arc.

To find the radius of such an arc and thus the radius of the curvature of the curve at a point x_0 , the normal in x_0 should be intersected with the normal in $x_0 + \epsilon$. This yields the x value of the center of the curvature circle M of the curve. The following drawing is intended to illustrate this.



$$x = x_0 - \frac{f'(x_0)}{f''(x_0)} - \frac{[f'(x_0)]^3}{f''(x_0)} = x_0 + \underbrace{\left[-\frac{f'(x_0)}{f''(x_0)} \cdot \left\{1 + [f'(x_0)]^2\right\}\right]}_{\Delta x_m}$$

For the corresponding change Δy_m of the ordinate y, we multiply the slope of the normal with Δx_m :

$$\Delta y_m = -\frac{1}{f'(x_0)} \cdot \Delta x_m = \frac{1}{f''(x_0)} \cdot \left\{ 1 + \left[f'(x_0) \right]^2 \right\}$$

With the Pythagorean theorem we get for the curvature radius:

$$\rho = \sqrt{(\Delta x_m)^2 + (\Delta y_m)^2} = \sqrt{(\Delta x_m)^2 + \left[-\frac{1}{f'(x_0)} \cdot \Delta x_m\right]^2} = \left|\frac{\Delta x_m}{f'(x_0)}\right| \cdot \sqrt{[f'(x_0)]^2 + 1}$$

Using $\Delta x_m = -\frac{f'(x_0)}{f''(x_0)} \cdot \left\{1 + \left[f'(x_0)\right]^2\right\}$ — this leads to:

$$\rho = \left| \frac{\frac{f'(x_0)}{f''(x_0)} \cdot \left\{ 1 + [f'(x_0)]^2 \right\}}{f'(x_0)} \right| \cdot \sqrt{[f'(x_0)]^2 + 1} = \frac{\sqrt{\left\{ 1 + [f'(x_0)]^2 \right\}^3}}{|f''(x_0)|}$$

1.5 Rolling without slipping



The condition of a rolling wheel without slipping forces, that the center of the wheel needs to make a rotation around the point P. Therefore, the center moves with the velocity:

 $\vec{v_{c}} = r \cdot \dot{\varphi} \cdot \vec{e_{t}}$ with normed tangent vector $\vec{e_{t}}$

Cause the center of the wheel also moves along the circle around M_c with radius $\rho - r$ and therefore the point P moves through the distance Δs to the point P'—the velocities in M_w and in P behave like their corresponding radii:

$$\vec{v_{c}} = \frac{\rho - r}{\rho} \cdot \frac{\Delta s}{\Delta t} \cdot \vec{e_{t}}$$
 with very small intervals, thus $\frac{\Delta s}{\Delta t} = \dot{s}$

Equating the right sides of both equations for the velocity of the center of the wheel finally leads to:

$$r \cdot \dot{\varphi} = \frac{\rho - r}{\rho} \cdot \dot{s} \quad \Rightarrow \quad \frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{\rho - r}{\rho \cdot r} \cdot \frac{\mathrm{d}s}{\mathrm{d}t} \quad \Rightarrow \quad \mathrm{d}\varphi = \frac{\rho - r}{\rho \cdot r} \cdot \mathrm{d}s = \frac{\rho - r}{\rho \cdot r} \cdot \sqrt{1 + [f'(x)]^2} \cdot \mathrm{d}x$$

2 Predefined vehicles

This package contains a number of predefined vehicles, like *Bike*, *Tractor*, *Highwheeler*, *Truck*, *Segway*, *Unicycle*. The last two of the vehicles only contain one axis, the rest has two axes.

Except the mono-cycles, a vehicle is defined by the radius of each wheel, [rB] for the rear (back) wheel and [rF] for the front wheel and the distance [d] between the axes of the two wheels. Their values must be given within the options of the \psVehicle [options] command. The cladding of a vehicle, auto body or bicycle frame must of course be adapted to the dimensions indicated above. A number of types of wheels and vehicles have been predefined.

We also setup some \newpsstyle for each of the vehicles, where the dimensions and the choice of the wheels are setup like we would choose them.

```
\newpsstyle{segway}{rB=1.4,backwheel=\segWheel}%MonoAxis
\newpsstyle{unicycle}{rB=1.6,backwheel=\SpokesWheelB}%MonoAxis
\newpsstyle{tractor}{d=4,rB=1.4,rF=1.0}
\newpsstyle{truck}{backwheel=\TruckWheel,frontwheel=\TruckWheel,d=6.28,rB=1.9,rF=1.9}
\newpsstyle{bike}{backwheel=\SpokesWheelB,frontwheel=\SpokesWheelB,d=5.8,rB=1.6,rF=1.6}
```

Here follows a list of the vehicles that come along with the package:





3 Predefined wheels

In this section we present 12 predefined wheels that can be used for the front and/or the back wheel.

3.1 \wheelA



\begin{pspicture}(-1,-1)(1,1)
\rput(!/rB 1 def 0 0){\wheelA}
\end{pspicture}

3.2 \wheelB



\begin{pspicture}(-1,-1)(1,1)
\rput(!/rB 1 def 0 0){\wheelB}
\end{pspicture}

3.3 \wheelC



\begin{pspicture}(-1,-1)(1,1)
\rput(!/rB 1 def 0 0){\wheelC}
\end{pspicture}

3.4 \wheelD



\begin{pspicture}(-1,-1)(1,1)
\rput(!/rB 1 def 0 0){\wheelD}
\end{pspicture}

3.5 \arrowWheel



\begin{pspicture}(-1,-1)(1,1)
\rput(!/rB 1 def 0 0){\arrowWheel}
\end{pspicture}

3.6 \TruckWheel



\begin{pspicture}(-1,-1)(1,1)
\rput(!/rB 1 def 0 0){\TruckWheel}
\end{pspicture}

3.7 \segWheel



\begin{pspicture}(-1,-1)(1,1)
\rput(!/rB 1 def 0 0){\segWheel}
\end{pspicture}

3.8 \SpokesWheelCrossed



\begin{pspicture}(-1,-1)(1,1)
\rput(!/rB 1 def 0 0){\SpokesWheelCrossed}
\end{pspicture}

3.9 \SpokesWheelA



\begin{pspicture}(-1,-1)(1,1)
\rput(!/rB 1 def 0 0){\SpokesWheelA}
\end{pspicture}

3.10 \SpokesWheelB



\begin{pspicture}(-1,-1)(1,1)
\rput(!/rB 1 def 0 0){\SpokesWheelB}
\end{pspicture}

3.11 \TractorFrontWheel



\begin{pspicture}(-1,-1)(1,1)
\rput(!/rF 1 def 0 0){\TractorFrontWheel}
\end{pspicture}

3.12 \TractorRearWheel



\begin{pspicture}(-1,-1)(1,1)
\rput(!/rB 1 def 0 0){\TractorRearWheel}
\end{pspicture}

4 How to use the command

This package offers the following command:

\psVehicle [Options] {scaling factor}{abscissa back wheel}{equation function}

Important note: The function has to be given in algebraic notation and not in RPN. The package pst-vehicle contains the options epsilon=, rB=, rF=, d=, gang=, vehicle=, ownvehicle=, backwheel=, frontwheel=, MonoAxis=, showSlope= and startPos=

Name	Default	Meaning	
epsilon	1e-6	Increment	
rB	1.6	Radius of the back wheel	
rF	1.6	Radius of the front wheel	
d	5.8	Distance between front and back wheel axes	
gang	1	Transmission between pedals and back wheel	
vehicle	\Bike	The Bike is chosen by default	
ownvehicle		Used to generate custom vehicle	
backwheel	\wheelA	wheelA is chosen by default	
frontwheel	\wheelA	wheelA is chosen by default	
MonoAxis	false	if the vehicle has one axis	
showSlope	true	showing the slope of the vehicle	
startPos	Θ	synchronizing the initial rotation of the wheels at the start	
		point	
GravNode	dA12 2 div 1	sets a node near center of gravity by default with name	
		GravC	

5 The Slope-o-Meter

A very nice gadget to show the angle of the slope of the vehicle on the curve. A fine thing for animations as you will see within the section Animation \dots

This command is shipped with two arguments to customize it with the *color of appearance* and the *angle of the pointer*.

Note: The name *Slope-o-Meter* is not at all an academically correct notation, but we all together had great fun to give it that special name.



\begin{pspicture}(-2.5,-2.5)(2.5,2.5)
\pstVerb{/omega 30 def}
\rput(0,0){\SlopeoMeter{cyan!90}{omega}}
\end{pspicture}

6 Examples

6.1 Predefined vehicle with custom wheels



6.2 Custom vehicle

To design your own vehicle, there are just a few rules to follow:

- Choose vehicle=\SelfDefinedVehicle
- No need to draw the wheels—just choose one of the predefined—or define your own wheels and then choose them with the options backwheel= and frontwheel=
- **Important note:** The axe of the back wheel is hard programmed at: O(0|0)
- The front wheel position is automatically calculated with the given distance between the two axes d= $% \left(\frac{1}{2}\right) =0$
- Draw your vehicle as if it stands on a horizontal plane, then define it and set it with i.e. ownvehicle=\myVeh as shown in the example below.



The same body of the vehicle is chosen as within the example above, but the front wheel has a smaller radius.



7 Animation

In case of animation there is a thing to be said: If tex memory is exceeded, please increase your memory. For T_EXLive users this can be done within the texmf.cnf. We chose the following:

main_memory = 12000000 % words of inimemory available; also applies to inimf&mp extra_mem_top = 60000000 % extra high memory for chars, tokens, etc. extra_mem_bot = 12000000 % extra low memory for boxes, glue, breakpoints, etc.

```
\def\funkg{0.25*(x-3)*sin(0.2*(x-2))-1}
\begin{animateinline}[controls,palindrome,
    begin={\begin{pspicture}(-2,-2)(13,3)},
    end={\end{pspicture}]{20}% 20 frames/s (velocity of the animation)
\multiframe{100}{rB=0+0.05}{% number of frames
\psframe*[linecolor=cyan!20](-2,-2)(13,4)
\pscustom[fillstyle=solid,fillcolor={[RGB]{174 137 100}},linestyle=none]{
\psplot[plotpoints=500,algebraic]{-2}{13}{\funkg}
\psline(13,-2)(-2,-2)
\closepath}
\psplot[plotpoints=500,algebraic]{-2}{13}{\funkg}
\psvehicle[vehicle=\Bike,style=bike,linecolor=DodgerBlue4]{0.4}{\rB}{\funkg}
\rput(10.5,0.5){\SlopeoMeter{cyan!90}{omega}}
}
\end{animateinline}
```

8 List of all optional arguments for pst-vehicle

Key	Туре	Default
backwheel	ordinary	\wheelA
frontwheel	ordinary	\wheelA
ownvehicle	ordinary	[none]
vehicle	ordinary	\Bike
GravNode	ordinary	
d	ordinary	5.8
rF	ordinary	1.6
rB	ordinary	1.6
gang	ordinary	1
epsilon	ordinary	1e-6
startPos	ordinary	Θ
showSlope	boolean	true
MonoAxis	boolean	false

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